Abstracts of invited speakers' talks

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Graphs and classes groups

A. Ballester-Bolinches

If $\mathfrak{X}$ is a class of groups, Delizia, Moravec and Nicotera (Bull. Austral. Math. Soc. 75, 313-320, 2007) call a group G $\mathfrak{X}$-transitive if whenever $\langle a, b\rangle$ and $\langle b, c\rangle$ are in $\mathfrak{X}$ $\langle a, c\rangle$ is also in $\mathfrak{X}$ $(a, b, c \in G)$. The structure of $\mathfrak{X}\mathfrak{T}$-groups has been investigated for a number of classes of groups, by Delizia, Moravec and Nicotera and others. A graph can be associated with a group in many ways. Delizia, Moravec and Nicotera introduce a graph which is a generalisation of the commuting graph of a group, but do not make use of the graph. In the first part of the talk, we will use the properties of the graph to investigate further classes of groups and to obtain more detailed structural information. In the second part of the talk a graph characterisation of the finite groups in which permutability is transitive is presented.

Regular groups, radical rings, and Abelian Hopf Galois structures on prime-power Galois field extensions

A. Caranti

To be communicated.

Discrete dynamical systems in group theory

D. Dikranjan

A discrete dynamical system in a category $\mathfrak{X}$ is an object X of $\mathfrak{X}$ provided with an endomorphism $T : X \to X$ in $\mathfrak{X}$. In most of the cases $\mathfrak{X}$ will be the category of (topological) groups and (continuous) group homomorphisms, the category of right modules over a ring R and the R-module homomorphisms, or just the category of topological (resp., measure) spaces and continuous (resp., measure preserving) maps. An isomorphism between two such systems $T : X \to X$ and $S : Y \to Y$ is an isomorphism $\xi : X \to Y$ in $\mathfrak{X}$ such that $\xi^{-1} \circ S \circ \xi = T$.
A fundamental numerical invariant used to classify the discrete dynamical systems up to isomorphism is the entropy. It was introduced in ergodic theory by Kolmogorov

and Sinai in 1958, and in topological dynamics by Adler, Konheim, and McAndrew [1]. These authors proposed also a brief general scheme for defining *algebraic* entropy in the context of abelian groups, developed further in [20, 5]. Since this approach was appropriate only for torsion groups, a modification was proposed by Peters [14] in the case of non-torsion abelian groups. A second modification was proposed in [2], since Peters' approach works only for monomorphisms. This notion of algebraic entropy h of arbitrary endomorphisms of abelian groups will be one of the main topics of these three lectures. Adjoint (dual) entropy in abelian groups was introduced in [4],[12]. The algebraic entropy was extended to the context of modules by Salce and Zanardo [16]. In all these cases the entropy is intended to measure the "chaos" or "disorder" created by the discrete dynamical system. Recently Salce, Vamos and Virili [15] found a fruitful connection between the algebraic entropy of module endomorphisms and multiplicities of length functions defined by Vámos in the sixties [17, 18].
The aim of the first lecture is to expose the unifying approach from [6] that will allow us to obtain all notions of entropy mentioned above by using a single one $h_{\mathfrak{S}}$, defined for endomorphisms in a sufficiently simple category, namely the category $\mathfrak{S}$ of *normed commutative semigroups*. Once the entropy $h_{\mathfrak{S}}$ is defined, one can easily build a natural functor F from each of the above mentioned categories $\mathfrak{X}$ (assigning an appropriate normed semigroup FX to every object X of $\mathfrak{X}$), so that the specific entropy of a self-map T in $\mathfrak{X}$ can be obtained as the entropy $h_{\mathfrak{S}}(FT)$ in $\mathfrak{S}$. This approach simultaneously covers the existing notions of entropy in the various categories [1, 2, 4, 5, 12, 10, 14, 15, 16, 20] and allows for a transparent uniform treatment of all these notions of entropy.
The second lecture is entirely dedicated to the algebraic entropy h in the category of abelian groups. The right Bernouli shift β_p of the group $\bigoplus_{\mathbb{N}} \mathbb{Z}_p$ has algebraic entropy $h(\beta_p) = \log p$. It turned out that the computation of the algebraic entropy of an endomorpfism ϕ of the group $\mathbb{Q}^n$ $(n \geq 1)$ is rather non-trivial. Let $f(x) = sx^n + a_1x^{n-1} + \ldots + a_n$ be the characteristic polynomial of ϕ written as a primitive polynomial over $\mathbb{Z}$. Then $h(\phi)$ coincides with the (logarithmic) Mahler measure $m(f) = \log s + \sum_{|\lambda_i|>1} \log |\lambda_i|$ of $f(x)$, where λ_i are the eigenvalues of $f(x)$ [7]. This fact, recently established by Giordano Bruno and Virili [11], know as *Algebraic Yuzvinskiĭ formula*, plays a crucial role in understanding the algebraic entropy. It truns out, that this formula, along with the equalities $h(\beta_p) = \log p$ for each prime p, and other three natural properties of h (namely, invariance of h under conjugation, the "Addition Theorem" for h and the "continuity" of h with respect to direct limits) determine uniquely the algebraic entropy. It is worth mentioning that the Algebraic Yuzvinskiĭ formula provides a remarkable connection between the algebraic entropy and the celebrated eighty years old Lehmer's problem on prime numbers ([13]). As another application of the Algebraic Yuzvinskiĭ formula we deduce an extension of Peters' theorem about the connection between the algebraic entropy of an endomorphism $\phi : G \to G$ and the topological entropy of its Pontryagin dual $\widehat{\phi} : \widehat{G} \to \widehat{G}$.
The last lecture concerns the connection between the algebraic entropy and other

dynamical aspects of the endomorphisms of the abelian groups, for example periodic and quasi-periodic points, the dychotomy between polynomial and exponential growth of the orbits, etc. A relevant tool here is the *Pinsker subgroup* of a discrete dynamical system $\phi : G \to G$, namely the largest ϕ-invariant subgroup $\mathbf{P}(G, \phi)$ of G where the restriction of ϕ has entropy 0. It turns out that $\mathbf{P}(G, \phi)$ is also the largest ϕ-invariant subgroup of G where the restriction of ϕ has polynomial growth, as well as the smallest ϕ-invariant subgroup of G such that the induced endomophism $\bar{\phi} : G/\mathbf{P}(G, \phi) \to G/\mathbf{P}(G, \phi)$ has no quasi-periodic points.

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Dynkin Diagrams, Support Spaces and Representation Type

R. Farnsteiner

The category of finite-dimensional associative algebras over an algebraically closed field can be subdivided via the notion of representation type. Algebras of finite representation type only possess finitely many isomorphism classes of finite-dimensional indecomposable modules. For representation-infinite algebras, one distinguishes between tame and wild type. While the isomorphism classes of indecomposable modules over tame algebras occur in each dimension in at most finitely many one-parameter families (and thus can usually be classified), the presence of a two-parameter family renders such a classification a wild problem.
In these lectures, I will explain how a combination of techniques from the representation theory of quivers and geometric methods involving cohomological support varieties leads to an understanding of the aforementioned subdivision for cocommutative Hopf algebras. In this context, Dynkin diagrams appear via a connection between Ext-quivers of associative algebras and McKay quivers of finite groups.

Generalisations of Finite T-groups

A. D. Feldman

This talk will discuss work with Adolfo Ballester-Bolinches, James Beidleman, M.C. Pedraza-Aguilera, and M. F. Ragland. Let f be a subgroup embedding function such that for every finite group G, $f(G)$ contains the set of normal subgroups of G and is contained in the set of Sylow-permutable subgroups of G. We say $H\ f\ G$ if H is an element of $f(G)$. Given such an f, let fT denote the class of finite groups in which $H\ f\ G$ if and only if H is subnormal in G; because Sylow-permutable subgroups are subnormal, this is the class in which f is a transitive relation. Thus if $f(G)$ is, respectively, the set of normal subgroups, permutable subgroups, or Sylow-permutable subgroups of G, then fT is, respectively, the class of T-groups, PT-groups, or PST-groups. Let $\mathcal{F}$ be a formation of finite groups containing all nilpotent groups such that any normal subgroup of any fT-group in $\mathcal{F}$ and any subgroup of any soluble fT-group in $\mathcal{F}$ belongs to $\mathcal{F}$. A subgroup M of a finite group G is said to be $\mathcal{F}$-normal in G if $G/Core_G(M)$ belongs to $\mathcal{F}$. A subgroup U of a finite group G is called a K-$\mathcal{F}$-subnormal subgroup of G if either $U = G$ or there exist subgroups $U = U_0 \leq U_1 \leq \cdots \leq U_n = G$ such that U_{i-1} is either normal or $\mathcal{F}$-normal in U_i, for $i = 1, 2, \ldots, n$. We call a finite group G an $fT_{\mathcal{F}}$-group if every K-$\mathcal{F}$-subnormal subgroup of G is in $f(G)$. When $\mathcal{F}$ is the class of all finite nilpotent groups, the $fT_{\mathcal{F}}$-groups are precisely the fT-groups. We analyse the structure of $fT_{\mathcal{F}}$-groups, particularly where the fT-groups are the T-, PT-, and PST-groups.

Some trends in the theory of groups with restricted conjugacy classes

F. de Giovanni

To be communicated.

Rational representations and units of integral group rings of finite groups

E. Jespers

The integral group ring $\mathbb{Z}G$ of a finite group is a ring of fundamental interest that gives an obvious link between group and ring theory. In this context, its group of invertible elements $\mathcal{U}(\mathbb{Z}G)$ is an object of crucial importance. Even after the discovery of a counter example to the integral isomorphism problem, by Hertweck, one needs to continue the study of interesting special cases and to refine our understanding of the structure of the unit group $\mathcal{U}(\mathbb{Z}G)$. In this series of three lectures we focus on one of the problems posed by S.K. Sehgal: give a presentation by generators and relations for $\mathcal{U}(\mathbb{Z}G)$ for some finite groups.
The aim of the lectures is three fold: (1) present the needed back ground in order to tackle this problem, (2) survey some known results, (3) present new results.
The outline of the lecture is as follows.

1. Introduction and Motivation
2. Orders
3. Finite Unit Groups
4. Commutative Orders and Central Units
5. Non-commutative group rings and large unit groups
6. Rational representations of nilpotent groups
7. Applications to unit groups
8. Exceptional Simple Components
9. Structure theorems

Lecture 1 will deal with topics 1-4: hence dealing with the central part of the unit group. Lecture 2 with topics 5-7: a reduction of the study of $\mathcal{U}(\mathbb{Z}G)$ to central units and special linear groups and construction of generators provided some representations do not occur. Lecture 3 covers topics 8-9: a description of the exceptional representations and a structure theorem in case of the exceptional representations.

Representation growth and zeta functions of groups

B. Klopsch

In my talk I will give a short introduction to the subject of representation growth and zeta functions of groups. Subsequently I will report on recent results in this area, in particular from a series of joint papers with Avni, Onn and Voll on representation zeta functions of arithmetic groups. A group G is said to be (representation) rigid, if for every positive integer n the number $r_n(G)$ of complex linear representations of G of dimension n is finite. Of particular interest are groups G for which the total number of representations up to degree N, viz. $R_N(G) = \sum_{n=1}^{N} r_n(G)$, is bounded by a polynomial in N. For such groups G the arithmetic sequence $r_n(G)$ is encoded in a Dirichlet generating function, the representation zeta function $\zeta_G(s) = \sum_{n=1}^{\infty} r_n(G)n^{-s}$ where s is a complex variable. A key invariant of the Dirichlet series $\zeta_G(s)$ is its abscissa of convergence $\alpha(G)$: it provides the precise polynomial degree of growth in the total number of representations $R_N(G)$ as $N \to \infty$. A central conjecture in the area, put forward by Larsen and Lubotzky, predicts that any two arithmetic lattices Γ_1 and Γ_2 in a higher rank semisimple group H have the same degree of representation growth: $\alpha(\Gamma_1) = \alpha(\Gamma_2)$. This can be regarded as a quantitative refinement of the well known Congruence Subgroup Conjecture of Serre.

Automorphisms of group extensions

D. J. S. Robinson

To be communicated.

$*$-Group identities in Units of group algebras S. Sehgal

Analogous to Amitsur's $*$-identities in rings we introduce the concept of $*$-group identities in $U(FG)$. We classify torsion groups so that the unit group of their group algebras satisfy a $*$-group identity . The history and motivation will be given for such an investigation.

Products of groups and Yang-Baxter equations

Y. Sysak

To be communicated.

Commutator width in Chevalley groups

N. Vavilov

The talk we present some recent result by Alexei Stepanov, and the author on commutators and commutator width in Chevalley groups over rings. Morally, they go in the direction opposite to Ore's conjecture, and amount to saying that a Chevalley group over a general ring has very few commutators. This is culminated by the result asserting that the width of commutators in elementary generators is bounded by a universal constant, depending on type of the group alone.
Also, we discuss the underlying methods (mostly, versions of localisation methods, including some new ones), some further related results on commutators, and on bounded generation and factorisations of Chevalley groups. In fact, many of our results are already new for the special linear group $SL(n, R)$ and even for finite fields give better bounds, than the known ones.